11 FUNCTIONS

Functions arise whenever one quantity depends on another.

(Stewart CALCULUS)

Example: The human population of the world $P$ depends on the time $t$.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>3040</td>
</tr>
<tr>
<td>1970</td>
<td>3710</td>
</tr>
<tr>
<td>1980</td>
<td>4450</td>
</tr>
<tr>
<td>1990</td>
<td>5280</td>
</tr>
<tr>
<td>2000</td>
<td>6080</td>
</tr>
<tr>
<td>2010</td>
<td>6870</td>
</tr>
</tbody>
</table>

For each value of the time $t$, there is a corresponding value of $P$, we say that $P$ is a function of $t$.

We can think of a function as a kind of machine.

We feed the machine with input $x$, and the function $f$ returns it to a "single" output $f(x)$. 
Understand the mystery function rule:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>

"The function rule: Multiply by 5!"

WHAT'S MY RULE?
Class Activity: Create your function table for your classmates to solve.

Functions in the Real World:

• Put money in a vending machine, make a selection, and a specific item drops into the output slot.
• Plant growth depends on sunlight and rainfall (water).
• Speed depends on the distance travelled and time taken.
• A weekly salary is a function of the hourly pay rate and the number of hours worked.
• The length of a shadow is a function of the height and the time of the day.
HOW TO REPRESENT A FUNCTION:

1. Rule of the function (FORMULA)

\[ f(x) = \sqrt{x} \]

2. Arrow diagram

3. Graph

If \( f \) is a function with domain \( D \), then its graph is the set of ordered pairs

\[ \{(x, f(x)) \mid x \in D\} \]

The graph of \( f \) consists of all points \((x, y)\) in the coordinate plane such that \( y = f(x) \) and \( x \in D \).
Domain and Range:

The domain of a function is the set of values of the independent variable (input) for which the function is defined.

The range of a function is the set of all "output values" of the function.

Example: The domain of the function \( f(x) = x^2 \) is the set of all real numbers.

Example: The domain of the function \( g(x) = \sqrt{x} \) is the set of all non-negative real numbers.

Example: The domain of \( h(x) = \frac{1}{x} \) is the set of all nonzero real numbers.

Example: The range of \( f(x) = x^2 \) is the set of all non-negative real numbers.
4. Table

Example: A table of values for the population size of a city during different years:

<table>
<thead>
<tr>
<th>Years (t)</th>
<th>Population (P(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>1695</td>
</tr>
<tr>
<td>1986</td>
<td>1716</td>
</tr>
<tr>
<td>1988</td>
<td>2100</td>
</tr>
</tbody>
</table>

**Vertical Line Test**: A curve in the xy-plane is the graph of a function of x iff no vertical line intersects the curve more than once.

**Example**: \( f(x) = x^2 \)

No matter where we drop a vertical line, it only hits the parabola in one spot.

\[ \Rightarrow \text{It's a function.} \]
0. **Sideways Parabola**

\[ \rightarrow \text{It's not a function} \]

0. **Linear function**

\[ \rightarrow \text{It's a function} \]

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**Essential Functions**

1. **Constant function**;

\[ f(x) = c \]

It disregards the input and always outputs the constant \( c \).
2. Linear function:

\[ f(x) = mx + c \]

\[ f(x) = 2x + 3 \]

3. Identity function:

\[ f(x) = x \]

Takes an input and outputs it unchanged

4. Quadratic function

\[ f(x) = ax^2 + bx + c \]

It's a polynomial of the second degree. Its graph is a parabola unless \( a = 0 \).
Piecewise defined functions;

Example: (STEWART CALCULUS)

A function \( f \) is defined by

\[
    f(x) = \begin{cases} 
        1-x & \text{if } x \leq -1 \\
        x^2 & \text{if } x > -1
    \end{cases}
\]

Evaluate \( f(-2), f(-1), f(0) \) and sketch the graph.

Solution:

\[
    f(-2) = 1 + 2 = 3 \\
    f(-1) = 1 + 1 = 2 \\
    f(0) = 0
\]
**Example:** Sketch the graph of the absolute value function $f(x) = |x|$

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$


- If $f$ satisfies $f(-x) = -f(x)$ for every number $x$ in its domain, then $f$ is called an **odd** function.
- If $f$ satisfies $f(-x) = f(x)$ for every number $x$ in its domain, then $f$ is called an **even** function.

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**Examples**

1) $f(x) = x^4 + x^2 \quad \rightarrow \quad \text{EVEN}$
2) $f(x) = 5x - x^2 \quad \rightarrow \quad \text{NEITHER}$
3) $f(x) = x^3 \quad \rightarrow \quad \text{ODD}$
4) $f(x) = x^5 + 7x \quad \rightarrow \quad \text{ODD}$
Increasing and Decreasing Functions

If \( x_1, x_2 \in \text{Domain of } f \)

\[ x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2), \text{ the function is said to be increasing.} \]

\[ x_1 \leq x_2 \Rightarrow f(x_1) > f(x_2), \text{ the function is said to be decreasing.} \]

Examples

1) \( y = x^2 \) for \( x > 0 \) is increasing.

2) \( y = 1 - x \) is decreasing.