

### Worksheet 5 - Antiderivative, Definite Integral (4.9,5.1.5.2)

1. Fill in the missing information to show that the area between the  $x$ -axis and the graph of  $f(x) = 3x + 10$  on the interval  $[2, 7]$  can be expressed as the limit of a right Riemann sum. The only variables appearing in your limit should be  $n$  and  $k$ . Do not evaluate this limit.

$$AREA = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[ \quad \quad \quad \right]$$

2. Fill in the missing information to show that the area between the  $x$ -axis and the graph of  $f(x) = 2x + 1$  on the interval  $[5, 8]$  can be expressed as the limit of a Left Riemann sum. The only variables appearing in your limit should be  $n$  and  $k$ . You do not need to evaluate this limit.

$$AREA = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[ \quad \quad \quad \right]$$

3. Evaluate the following limit.

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{5k}{n^3} + \frac{7}{n} \right)$$

4. The definite integral  $\int_2^6 e^{t^2} dt$  can be written as a limit. Fill in the missing information in this limit.

$$\int_2^6 e^{t^2} dt = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[ \quad \quad \quad \right]$$

5. Suppose that  $f$  is an odd function and  $g$  is an even function which are each integrable on the interval  $[-5, 5]$ . Given that  $\int_0^5 f(x) dx = 8$  and  $\int_0^5 g(x) dx = 3$ , evaluate the following definite integrals.

(a)  $\int_5^0 g(x) dx$

(b)  $\int_5^5 f(x) dx$

$$(c) \int_{-5}^5 (2f(x) + 4g(x)) dx$$

$$(d) \int_{-5}^5 (4 + (f(x))^3) dx$$

6. Find a formula for  $f(x)$  given that  $f''(x) = 5 \sin x + 3 \cos x$ ,  $f(0) = 10$ , and  $f'(0) = 10$ .
7. A function  $f(x)$  has derivative  $f'(x) = 6x^2 + 5$ . Find a formula for  $f(x)$  given that its graph goes through the point  $(1, 15)$ .
8. Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{14k}{n^2} - \frac{4}{n} \right)$$

9. Suppose that  $f$  is an odd function which is integrable on the interval  $[-5, 5]$ . If

$$\int_0^2 f(x) dx = 4 \quad \text{and} \quad \int_2^3 f(x) dx = 10, \quad \text{then evaluate the following quantities.}$$

$$(a) \int_0^5 f(x) dx + \int_5^3 f(x) dx$$

$$(b) \int_{-2}^2 f(x) dx$$

$$(c) \int_{-2}^2 f(|x|) dx$$