Worksheet

1. $\lim_{x\to\infty} \frac{10e^x+1}{6-2e^x} = \lim_{x\to\infty} \frac{e^x\left(10+\frac{1}{e^x}\right)}{e^x\left(\frac{6}{e^x}-2\right)} = \lim_{x\to\infty} \frac{10+\frac{1}{e^x}}{\frac{6}{e^x}-2} = -5$

($\frac{1}{e^x} \to 0$, $\frac{6}{e^x} \to 0$)

$$\lim_{x\to-\infty} \frac{10e^x+1}{6-2e^x} = \frac{10\cdot 0+1}{6-2\cdot 0} = \frac{1}{6}$$

$y=-5$ and $y=\frac{1}{6}$ are horizontal asymptotes

2. $F(x) = x^3 - 1 - x - x^2$

F is continuous on everywhere

$F(0) = -1 < 0$

$F(2) = 8-1-2-4 = 1 > 0$

F is continuous on $[0,2]$ and $F(0) < 0 < F(2)$

so there exist $c \in (0,2)$ such that $F(c)=0$

$\Rightarrow c^3 - 1 - c - c^2 = 0$

$\Rightarrow c^3 = 1 + c + c^2$

3. a) $\lim_{x \to e^+} \frac{x^2}{\underbrace{1-\ln x}_{\to 0^-}} = \frac{e^2}{0^-} = -\infty$

b) $\lim_{x \to 0} \frac{\sqrt{9+x} - 3}{2x} = \lim_{x \to 0} \frac{(\sqrt{9+x} - 3)(\sqrt{9+x} + 3)}{2x(\sqrt{9+x} + 3)}$

$= \lim_{x \to 0} \frac{9+x-9}{2x(\sqrt{9+x} + 3)} = \lim_{x \to 0} \frac{1}{2(\sqrt{9+x} + 3)}$

$= \frac{1}{2 \cdot 6} = \frac{1}{12}$

c) $\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{x(x-4)}{(x-4)(x+1)} = \frac{4}{5}$

d) $\lim_{t \to 0} \frac{1}{t} - \frac{1}{t^2+t} = \lim_{t \to 0} \frac{1}{t} - \frac{1}{t(t+1)}$

$= \lim_{t \to 0} \frac{t+1-1}{t(t+1)} = \lim_{t \to 0} \frac{t}{t(t+1)} = 1$

e) $\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} = f'(3)$ when $f(x) = \frac{1}{x}$

$f'(x) = \frac{-1}{x^2}$ $\quad f'(3) = \frac{-1}{9}$

$\Rightarrow \lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} = -\frac{1}{9}$

5. $x<2$ $f(x)$ is continuous (rational AND Denominator $\neq 0$)

$2<x<3$ $f(x)$ is continuous (Polynomial)

$x>3$ $f(x)$ is continuous (Polynomial)

We should check $x=2$ AND $x=3$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} x+2 = 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} ax^2-bx+3 = 4a-2b+3$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \text{ so that limit exist}$$

$$4 = 4a-2b+3$$

$$\boxed{1 = 4a-2b} \quad *$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x-a+b = 6-a+b$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} ax^2-bx+3 = 9a-3b+3$$

$$6-a+b = 9a-3b+3$$

$$\boxed{10a-4b=3} \quad **$$

$$-2 \quad 1 = 4a-2b$$

$$3 = 10a-4b$$

$$-2 = -8a+4b$$

$$3 = 10a-4b$$

$$1 = 2a$$

$$\boxed{a = 1/2}$$

$$1 = 4a-2b$$

$$1 = 2-2b$$

$$2b = 1$$

$$\boxed{b = 1/2}$$

6. $f(x) = \dfrac{x-3}{3x^2-8x-3} = \dfrac{x-3}{(3x+1)(x-3)}$

$3x^2-8x-3$

$3x \quad 1$

$x \quad -3$

$x \quad 1$

$f(x)$ is not defined at $x=3$, but

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \dfrac{(x-3)}{(3x+1)(x-3)} = \dfrac{1}{10}$ limit exist

f has "Removable Discontinuity" at $x=3$

$\lim_{x\to -\frac{1}{3}^-} \dfrac{x-3}{(3x+1)(x-3)} = \lim_{x\to -\frac{1}{3}^-} \dfrac{1}{(3x+1)} = -\infty$ ($3x+1 \to 0^-$)

f has "infinite Discontinuity" at $x = -1/3$

7. It's your Quiz Question (Check your Quiz Solution)

8. w is continuous at b

$\Rightarrow \lim_{x\to b} w(x) = w(b)$

9. Volume $= x(x+1)(x+2)$

Surface Area $2(x(x+1)+x(x+2)+(x+1)(x+2))$

$2(x^2+x+x^2+2x+x^2+3x+2)$

$2(3x^2+6x+2)$

$V =$ Surface Area

$x(x+1)(x+2) = 2(3x^2+6x+2)$

$x^3+3x^2+2x = 6x^2+12x+4$

$x^3-3x^2-10x-4=0$

$F(x)=x^3-3x^2-10x-4$

$F(0)=-4<0$

$F(6)=44>0$

F is polynomial so it is continuous on $[0,6]$

$F(0)<0<F(6)$

$\Rightarrow \exists\ c \in (0,6)$ s.t $F(c)=0$ □

10. You had same kind of question in your Test 1

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

You should apply this formula.

11. $y=x^4-8x^2+56$

$y'=4x^3-16x$ → slope of the tangent line at x

Horizontal → slope is 0

$4x^3-16x=0$

$4x(x^2-4)=0 \Rightarrow x=0,2,-2$

$4x(x-2)(x+2)=0$

12. $y=x^3+6x^2-5$ at $(1,2)$

$y'=3x^2+12x$

$y'(1)=15$ → Slope of the tangent line 15

Slope of the normal line $\frac{-1}{15}$

Equation of TANGENT $(y-2)=15(x-1)$

Equation of Normal $(y-2)=-\frac{1}{15}(x-1)$

13. You had same in your quiz. Please Check Quiz Solutions.

14. a) $f(x) = (x^5+30x-5).e^x$ (Product Rule)

$$f'(x) = (5x^4+30).e^x + (x^5+30x-5).e^x$$

b) $g(x) = \frac{x+2}{x^2+3x+4}$ (Quotient Rule)

$$g'(x) = \frac{(x+2)'(x^2+3x+4) - (x+2)(x^2+3x+4)'}{(x^2+3x+4)^2}$$

$$g'(x) = \frac{(x^2+3x+4) - (x+2).(2x+3)}{(x^2+3x+4)^2}$$

c) $g(t) = \frac{t-5\sqrt{t}}{t^{1/3}} = \frac{t}{t^{1/3}} - \frac{5\sqrt{t}}{t^{1/3}} = t^{\frac{2}{3}} - 5t^{1/2-1/3}$

$$= t^{2/3} - 5t^{1/6}$$

$$g'(t) = (t^{2/3})' - (5t^{1/6})'$$

$$= \frac{2}{3}t^{-1/3} - \frac{5}{6}t^{\frac{-5}{6}}$$

15. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$ → APPLY Quotient Rule

16. a) $f'(x) = (6x^3)' - (5\sin x)'$

$= 12x^2 - 5\cos x$

b) $y = \frac{1-\sec x}{\tan x} = \frac{1-\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{\cos x - 1}{\sin x}$

$y' = \left(\frac{\cos x - 1}{\sin x}\right)' = \frac{(\cos x - 1)'\sin x - (\cos x - 1)(\sin x)'}{\sin^2 x}$

$= \frac{-\sin x \sin x - (\cos x - 1)\cdot \cos x}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x + \cos x}{\sin^2 x}$

$= \frac{-1 + \cos x}{\sin^2 x} = \frac{-1+\cos x}{1-\cos^2 x}$

$= \frac{-1+\cos x}{(1-\cos x)(1+\cos x)}$ -1

$= \frac{-1}{1+\cos x}$ □

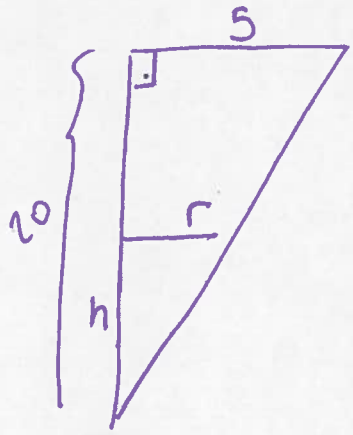
17 $y = x + \tan x$ at (π, π)

$y' = 1 + \sec^2 x$

$y'(\pi) = 1 + \frac{1}{\cos^2(\pi)} = 1 + 1 = 2$ ← slope

$$y - \pi = 2(x - \pi)$$

18.



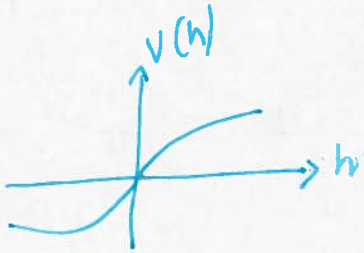
From the similar triangles

$\frac{r}{5} = \frac{h}{20} \Rightarrow r = \frac{h}{4}$

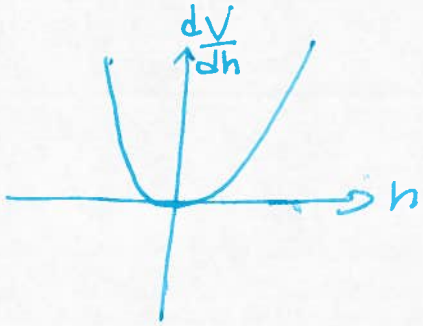
Volume $= \frac{1}{3}\pi r^2 \cdot h$

$V(h) = \frac{1}{3}\pi \cdot \frac{h^2}{4} \cdot h$

$V(h) = \frac{\pi}{12} h^3$



$\frac{dV}{dh} = \frac{\pi 3h^2}{12} = \frac{\pi h^2}{4} \Rightarrow$



19 $m(t)$ = concentration of $[N_2O_5]$

a) $\Rightarrow \frac{dm}{dt} = -0.0005 \cdot m(t)$

$\Rightarrow m(t) = \frac{C}{\text{initial mass}} \cdot e^{-0.0005 \cdot t}$

b) $m(t) = C \cdot \frac{90}{100}$

$\Rightarrow m(t) = C \cdot \frac{9}{10}$

$C \cdot e^{-0.0005 \cdot t} = C \cdot \frac{9}{10}$

$e^{-0.0005 \cdot t} = 0.9$

Take ln $\Rightarrow -0.0005t = \ln(0.9)$

$$\boxed{t = -\frac{\ln(0.9)}{0.0005}}$$

20.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{dR_1}{dt} = 0.3\ \Omega/s \qquad \frac{dR_2}{dt} = 0.2\ \Omega/s$$

Question $\left.\frac{dR}{dt}\right|_{R_1 = 80\Omega,\ R_2 = 100\Omega}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R^{-1} = R_1^{-1} + R_2^{-1}$$

$$-\frac{1}{R^2}\cdot\frac{dR}{dt} = -\frac{1}{R_1}\cdot\frac{dR_1}{dt} - \frac{1}{R_2}\cdot\frac{dR_2}{dt}$$

$$-\frac{1}{\left(\frac{400}{9}\right)^2}\cdot\frac{dR}{dt} = \frac{-1}{80}\cdot 0.3 - \frac{1}{100}\cdot 0.2$$

$$\frac{dR}{dt} = -\left(\frac{400}{9}\right)^2\cdot\left(\frac{-0.3}{80} - \frac{0.2}{100}\right)$$

→ Calculator !!

We should calculate R for given R_1 and R_2

$$\frac{1}{R} = \frac{1}{80} + \frac{1}{100}$$

(5) (4)

$$\frac{1}{R} = \frac{5+4}{400} = \frac{9}{400}$$

$$\Rightarrow R = \frac{400}{9}$$