

MTH 220 Quiz 7

1. If $x^3 + y^3 = 4xy$, find $\frac{dy}{dx}$. Then equation of the tangent line to the $x^3 + y^3 = 4xy$ at the point (2,2). (4 points.)

Solution:

$$\begin{aligned}\frac{d}{dx}(x^3 + y^3) &= \frac{d}{dx}4xy \\ 3x^2 + 3y^2 \frac{dy}{dx} &= 4(y + x \cdot 1 \cdot \frac{dy}{dx}) \\ \frac{dy}{dx}(3y^2 - 4x) &= 4y - 3x^2 \\ \frac{dy}{dx} &= \frac{4y - 3x^2}{3y^2 - 4x}\end{aligned}$$

Slope of the tangent line at (2,2) is $\frac{dy}{dx}(2, 2) = \frac{8-12}{12-8} = -1$

Equation of the tangent line at (2,2) is $y - 2 = -1(x - 2)$, which is $y = -x + 4$

2. Find the following derivatives (2 points each.)

(a) $f(x) = (x^5 + 4x^2 + 3)^5$

Solution:

$$f'(x) = 5(x^5 + 4x^2 + 3)^4 \cdot (x^5 + 4x^2 + 3)' = 5(x^5 + 4x^2 + 3)^4 \cdot (5x^4 + 8x)$$

(b) $g(x) = \ln(\sec x)$

Solution:

$$g'(x) = \frac{(\sec x)'}{\sec x} = \frac{\sec x \cdot \tan x}{\sec x} = \tan x$$

(c) $e^{(\sin 2x)}$

Solution: $(e^{\sin(2x)})' = (e^{\sin(2x)}) \cdot (\sin 2x)' = (e^{\sin(2x)}) \cdot \cos(2x) \cdot 2$